Topology-dependent rationality and quantal response equilibria in structured populations

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Nash equilibria of games are frequently used to reason about the decision-making. However, the underlying assumption of perfect rationality has been shown to be violated in many examples of decision-making in the real world. Accordingly, we explore a graded notion of rationality in socio-ecological systems of networked actors. We parametrise an actors' rationality via their place in a social network and quantify system rationality via the average Jensen-Shannon divergence between the games' Nash and Logit Quantal Response equilibria [1].

Previous work by [2] has argued that scale-free topologies maximise a system's overall rationality in this setup. Here, we show that while, for certain games, it is true that increasing degree heterogeneity of complex networks enhances rationality, rationality-optimal configurations are not scale-free. For the Prisoner's Dilemma and Stag Hunt games, we provide analytic arguments complemented by numerical optimisation experiments to demonstrate that core-periphery networks composed of a few dominant hub nodes surrounded by a periphery of very low degree nodes give strikingly smaller overall deviations from rationality than scale-free networks. If isolated nodes are allowed to form during optimisation, optimal networks are found to consist of a core made up by a complete graph with all other nodes being isolated. Similarly, for the Battle of the Sexes and the Matching Pennies games, we find that the optimal network structure is also a core-periphery graph but with a smaller difference in the average degrees of the core and the periphery. If no connectivity constraints are enforced, then in the case of the Battle of the Sexes a graph with a strongly bi-modal degree distribution emerges, while for the Matching Pennies game we obtain a quasi-regular graph. So, in contrast to [2], we have demonstrated that highly heterogeneous degree distributions do not necessarily maximise system rationality for all classes of games.

These results provide insight on the interplay between the topological structure of socio-ecological systems and their collective cognitive behaviour, with potential applications to understanding wealth inequality and the structural features of the network of global corporate control [3].

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